BOA: Branches and Binary Operators

Next, lets add

- Branches (`if` -expressions)
- Binary Operators (`+` , `-` , etc.)

In the process of doing so, we will learn about

- **Intermediate Forms**
- **Normalization**

Branches

Lets start first with branches (conditionals).

We will stick to our recipe of:

1. Build intuition with **examples**,
2. Model problem with **types**,
3. Implement with **type-transforming-functions**,
4. Validate with **tests**.

Examples

First, lets look at some examples of what we mean by branches.

- For now, lets treat `0` as "false" and non-zero as "true"

Example: `If1`

```
if 10:
  22
else:
  sub1(0)
```

- Since `10` is *not* `0` we evaluate the "then" case to get `22`

Example: `If2`

```
if sub(1):
  22
else:
  sub1(0)
```

- Since `sub(1)` *is* `0` we evaluate the "else" case to get `-1`

QUIZ: `If3`

~~DOG~~

`if-else` is also an *expression* so we can nest them:

What should the following evaluate to?

```
let x = if sub(1):
          22
        else:
          sub1(0)
in
  if x:
    add1(x)
  else:
    999
```

X86

"jump"

"branch" "comparisons"

"labels"

- **A.** `999`
- **B.** `0`
- **C.** `1`
- **D.** `1000`
- **E.** `-1`

Control Flow in Assembly

To compile branches, we will use **labels**, **comparisons** and **jumps**

Labels

```
our_code_label:
  ...
```

Labels are *"landmarks"*

- from which execution (control-flow) can be *started*, or
- to which it can be *diverted*

Comparisons

```
cmp a1, a2
```

at most one can be mem

- Perform a (numeric) **comparison** between the values `a1` and `a2` , and
- Store the result in a special **processor flag**

Jumps

```
jmp LABEL    # jump unconditionally (i.e. always)
je  LABEL    # jump if previous comparison result was EQUAL
jne LABEL    # jump if previous comparison result was NOT-EQUAL
```

Use the result of the **flag** set by the *most recent* `cmp`

- To *continue execution* from the given `LABEL`

QUIZ

Which of the following is a valid x86 encoding of

```
if 10:
  22
else:
  33
```

A

```
  mov rax, 10
  cmp rax, 0
  je  if_false
if_true:
  mov rax, 22
  jmp if_exit
if_false:
  mov rax, 33
if_exit:
```

B

```
  mov rax, 10
  cmp rax, 0
  je  if_false
if_true:
  mov rax, 22

if_false:
  mov rax, 33
if_exit:
```

C

```
  mov rax, 10
  cmp rax, 0
  je  if_true
if_true:
  mov rax, 22
  jmp if_exit
if_false:
  mov rax, 33
if_exit:
```

D

```
  mov rax, 10
  cmp rax, 0
  je  if_true
if_true:
  mov rax, 22

if_false:
  mov rax, 33
if_exit:
```

QUIZ: Compiling if-else

Strategy

To compile an expression of the form

```
if eCond:
  eThen
else:
  eElse
```

<< econd>>
cmp rax, 0
je IFFALSE
<<e then>>
jmp IFEXIT
IfFalse:
<< eElse>>
IfEXIT:

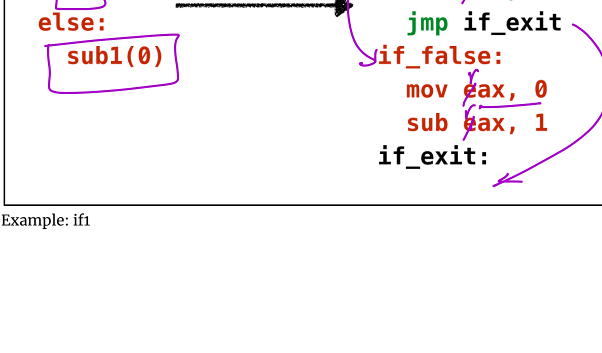
We will:

1. Compile `eCond`
2. Compare the result (in `rax`) against `0`
3. Jump if the result *is zero* to a **special** `"IfFalse"` label
   - At which we will evaluate `eElse` ,
   - Ending with a special `"IfExit"` label.
4. (Otherwise) continue to evaluate `eTrue`
   - And then jump (unconditionally) to the `"IfExit"` label.

Example: If-Expressions to `Asm`

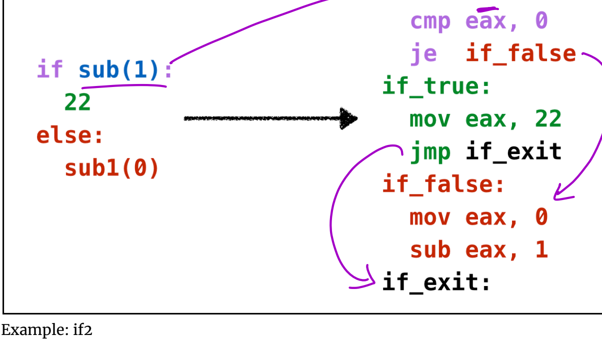
Lets see how our strategy works by example:

Example: if1



Example: if1

Example: if2



Example: if2

Example: if3

```
let x = if 10:                mov rax, 10
          22                  cmp rax, 0
        else:                 je  if_false
          0                   mov rax, 22
in                            jmp if_exit
  if x:                     if_false:
    55                        mov rax, 0
  else:                     if_exit:
    999                       mov [rsp - 8*1], rax
                              mov rax, [rsp - 8*1]
                              cmp rax, 0
                              je  if_false
                              mov rax, 55
                              jmp if_exit
                            if_false:
                              mov rax, 999
                            if_exit:
```

Example: if3

Oops, **cannot reuse labels** across if-expressions!

- Can't use same label in two places (invalid assembly)

```
let x = if 10:                mov rax, 10
          22                  cmp rax, 0
        else:                 je  if_false
          0                   mov rax, 22
in                            jmp if_exit
  if x:                     if_false:
    55                        mov rax, 0
  else:                     if_exit:
    999                       mov [rsp - 8*1], rax
                              mov rax, [rsp - 8*1]
                              cmp rax, 0
                              je  if_false
                              mov rax, 55
                              jmp if_exit
                            if_false:
                              mov rax, 999
                            if_exit:
```

Example: if3 wrong

Oops, need **distinct labels** for each branch!

- Require **distinct tags** for each `if-else` expression

```
let x = if 10:                mov rax, 10
        1 22                  cmp rax, 0
        else:                 je  if_1_false
          0                   mov rax, 22
in                            jmp if_1_exit
  if x:                     if_1_false:
  2 55                        mov rax, 0
  else:                     if_1_exit:
    999                       mov [rsp - 8*1], rax
                              mov rax, [rsp - 8*1]
                              cmp rax, 0
                              je  if_2_false
                              mov rax, 55
                              jmp if_2_exit
                            if_2_false:
                              mov rax, 999
                            if_2_exit:
```

Example: if3 tagged

Types: Source

Lets modify the *Source Expression* to add `if-else` expressions

```haskell
data Expr a
  = Number Int                      a
  | Add1   (Expr a)                 a
  | Sub1   (Expr a)                 a
  | Let    Id (Expr a) (Expr a)     a
  | Var    Id                       a
  | If     (Expr a) (Expr a) (Expr a) a
```

Polymorphic tags of type `a` for each sub-expression

- We can have *different types* of tags
- e.g. Source-Position information for error messages

Lets define a name for `Tag` (just integers).

```haskell
type Tag   = Int
```

We will now use:

```haskell
type BareE = Expr ()      -- AST after parsing
type TagE  = Expr Tag     -- AST with distinct tags
```

Types: Assembly

Now, lets extend the *Assembly* with labels, comparisons and jumps:

```haskell
data Label
  = BranchFalse Tag
  | BranchExit  Tag

data Instruction
  = ...
  | ICmp   Arg   Arg   -- Compare two arguments
  | ILabel Label       -- Create a label
  | IJmp   Label       -- Jump always
  | IJe    Label       -- Jump if equal
  | IJne   Label       -- Jump if not-equal
```

Transforms

We can't expect *programmer* to put in tags (yuck.)

- Lets squeeze in a `tagging` transform into our pipeline

Adding Tagging to the Compiler Pipeline

Transforms: Parse

Just as before, but now puts a dummy `()` into each position

```
λ> let parseStr s = fmap (const ()) (parse "" s)

λ> let e = parseStr "if 1: 22 else: 33"

λ> e
If (Number 1 ()) (Number 22 ()) (Number 33 ()) ()

λ> label e
If (Number 1 ((),0)) (Number 22 ((),1)) (Number 33 ((),2)) ((),3)
```

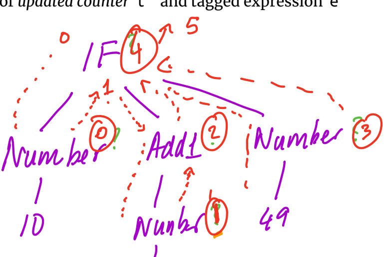
Transforms: Tag

doTag 0 e =

The key work is done by `doTag i e`

1. Recursively walk over the `BareE` named `e` starting tagging at counter `i`
2. Return a pair `(i', e')` of *updated counter* `i'` and tagged expression `e'`

if 10:
  add1(20)
else:
  49



QUIZ

```haskell
doTag :: Int -> BareE -> (Int, TagE)
doTag i (Number n _)     = (i + 1 , Number n i)
doTag i (Var    x _)     = (i + 1 , Var    x i)
doTag i (Let x e1 e2 _)  = (_2    , Let x e1' e2' i2)
  where
    (i1, e1')            = doTag i  e1
    (i2, e2')            = doTag _1 e2
```

i2+1

What expressions shall we fill in for `_1` and `_2` ?

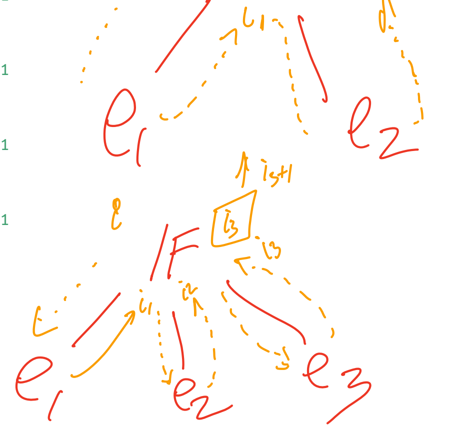
```haskell
{- A -}   _1 = i
          _2 = i + 1

{- B -}   _1 = i
          _2 = i1 + 1

{- C -}   _1 = i
          _2 = i2 + 1

{- D -}   _1 = i1
          _2 = i2 + 1

{- E -}   _1 = i2
          _2 = i1 + 1
```



(**ProTip:** Use `mapAccumL`)

We can now tag the whole program by

- Calling `doTag` with the initial counter (e.g. `0`),
- Throwing away the final counter.

```haskell
tag :: BareE -> TagE
tag e = e'  where  (_, e') = doTag 0 e
```

Transforms: Code Generation

Now that we have the tags we lets implement our compilation strategy

```haskell
compile env (If eCond eTrue eFalse i)
  =   compile env eCond ++             -- compile `eCond`
    [ ICmp (Reg RAX) (Const 0)         -- compare result to 0
    , IJe (BranchFalse i)              -- if-zero then jump to 'False'-block
    ]
   ++ compile env eTrue  ++            -- code for `True`-block
    [ IJmp   lExit      ]              -- jump to exit (skip `False`-block!)
   ++
      ILabel (BranchFalse i)           -- start of `False`-block
   : compile env eFalse ++             -- code for `False`-block
    [ ILabel (BranchExit i) ]          -- exit
```

False

Recap: Branches

- `Tag` each sub-expression,
- Use tag to generate control-flow labels implementing branch.

Lesson: Tagged program representation simplifies compilation…

- Next: another example of how intermediate representations help.

$e_1 + e_2$

Binary Operations

You know the drill.

1. Build intuition with **examples**,
2. Model problem with **types**,
3. Implement with **type-transforming-functions**,
4. Validate with **tests**.

Compiling Binary Operations

Lets look at some expressions and figure out how they would get compiled.

- Recall: We want the result to be in `rax` after the instructions finish.

QUIZ

What is the assembly corresponding to `33 - 10` ?

```
?1 rax, ?2
?3 rax, ?4
```

mov rax, 33
sub rax, 10

- **A.** `?1 = sub , ?2 = 33 , ?3 = mov , ?4 = 10`
- **B.** `?1 = mov , ?2 = 33 , ?3 = sub , ?4 = 10`
- **C.** `?1 = sub , ?2 = 10 , ?3 = mov , ?4 = 33`
- **D.** `?1 = mov , ?2 = 10 , ?3 = sub , ?4 = 33`

Example: Bin1

Lets start with some easy ones. The source:

```
2 + 3   ──────►   mov eax, 2
                  add eax, 3
```

Example: Bin 1

Strategy: Given `n1 + n2`

- Move `n1` into `rax` ,
- Add `n2` to `rax` .

$(((n_1 + n_2) + n_3) + n_4) + n_5$

mov rax, n_1
add rax, n_2
add rax, n_3
⋮

Example: Bin2

What if the first operand is a variable?

```
let x = 12          mov eax, 12
in        ──────►   mov [esp - 4], eax
    x + 10          mov eax, [esp - 4]
                    add eax, 10
```

Example: Bin 2

Simple, just copy the variable off the stack into `rax`

Strategy: Given `x + n`

- Move `x` (from stack) into `rax` ,
- Add `n` to `rax` .

$(((x_1 + x_2) + x_3) + \dots)$

mov rax, [RBP - 8*1]
add rax, [RBP - 8*2]
add rax, [RBP - 8*3]
⋮

Example: Bin3

Same thing works if the second operand is a variable.

```
                    mov eax, 12
let x = 12          mov [esp - 4], eax
  , y = 18  ──────► mov eax, 18
in                  mov [esp - 8], eax
    x + y           mov eax, [esp - 4]
                    add eax, [esp - 8]
```

Example: Bin 3

:%s/eax/rax/gc
:%s/esp/rbp/gc

Strategy: Given `x + n`

- Move `x` (from stack) into `rax` ,
- Add `n` to `rax` .

QUIZ

What is the assembly corresponding to `(10 + 20) * 30` ?

```
mov rax, 10
?1  rax, ?2
?3  rax, ?4
```

mov rax, 10
add rax, 20
mul rax, 3

- **A.** `?1 = add , ?2 = 30 , ?3 = mul , ?4 = 20`
- **B.** `?1 = mul , ?2 = 30 , ?3 = add , ?4 = 20`
- **C.** `?1 = add , ?2 = 20 , ?3 = mul , ?4 = 30`
- **D.** `?1 = mul , ?2 = 20 , ?3 = add , ?4 = 30`

Second Operand is Constant

In general, to compile `e + n` we can do

```
  compile e
++                   -- result of e is in rax
  [add rax, n]
```

e ⊕ x

((10 + 2) * 3) + 4

Bin
e ⊕ x
e ⊕ n
"immediate"

Example: Bin4

"BAD"

But what if we have *nested* expressions

```
(1 + 2) * (3 + 4)
```

rax

⇒ let t_1 = 1+2 ,
t_2 = 3+4
in $t_1 * t_2$

- Can compile `1 + 2` with result in `rax` ...
- .. but then need to *reuse* `rax` for `3 + 4`

Need to **save** `1 + 2` somewhere!

let t = 3+4
in
(1+2) * t

"GOOD"

Idea: How about use another register for `3 + 4`*?*

But then what about `(1 + 2) * (3 + 4) * (5 + 6)` ?

- In general, may need to *save* more sub-expressions than we have registers.

Question:

Why are `1 + 2` and `x + y` so easy to compile but `(1 + 2) * (3 + 4)` not?

Idea: Immediate Expressions

Why were `1 + 2` and `x + y` so easy to compile but `(1 + 2) * (3 + 4)` not?

As `1` and `x` are **immediate expressions**: their values don't require any computation!

- Either a **constant**, or,
- **variable** whose value is on the stack.

Idea: Administrative Normal Form (ANF)

An expression is in **Administrative Normal Form (ANF)**

> *ANF means all* ***primitive operations*** *have* ***immediate*** *arguments.*

Primitive Operations: Those whose values we *need* for computation to proceed.

- `v1 + v2`
- `v1 - v2`
- `v1 * v2`

QUIZ

> *ANF means all* ***primitive operations*** *have* ***immediate*** *arguments.*

Is the following expression in ANF?

```
(1 + 2) * (4 - 3)
```

A. Yes, its ANF.

B. Nope, its not, because of `+`

C. Nope, its not, because of `*`

D. Nope, its not, because of `-`

E. Huh, WTF is ANF?

Conversion to ANF

So, the below is *not* in ANF as `*` has **non-immediate** arguments

```
(1 + 2) * (4 - 3)
```

However, note the following variant *is* in ANF

```
let t1 = 1 + 2
  , t2 = 4 - 3
in
    t1 * t2
```

mov rax, 1
add rax, 2
mov [rbp-8], rax

mov rax, 4
sub rax, 3
mov [rbp-16], rax

mov rax, [rbp-8]
mul rax, [rbp-16]

How can we compile the above code?

```
; TODO in class
```

$V_1 * V_2$

Binary Operations: Strategy

We can convert *any* expression to ANF

- By adding "temporary" variables for sub-expressions

Text $\xrightarrow{\text{Parse}}$ AST $\xrightarrow{\text{Norm.}}$ ANF $\xrightarrow{\text{Tag}}$ ANF-Tag $\xrightarrow{\text{CodeGen}}$ ASM (.s)

Compiler Pipeline with ANF

- **Step 1:** Compiling ANF into Assembly
- **Step 2:** Converting Expressions into ANF

isANF :: Expr → Bool
isImm :: Expr → Bool

Types: Source

Lets add binary primitive operators

```
data Prim2
  = Plus | Minus | Times
```

and use them to extend the source language:

```
data Expr a
  = ...
  | Prim2 Prim2  (Expr a) (Expr a) a
```

So, for example, `2 + 3` would be parsed as:

```
Prim2 Plus (Number 2 ()) (Number 3 ()) ()
```

Types: Assembly

Need to add X86 instructions for primitive arithmetic:

```
data Instruction
  = ...
  | IAdd Arg Arg
  | ISub Arg Arg
  | IMul Arg Arg
```

Types: ANF

We *can* define a separate type for ANF (try it!)

... but ...

super tedious as it requires duplicating a bunch of code.

Instead, lets write a *function* that describes **immediate expressions**

```
isImm :: Expr a -> Bool
isImm (Number _ _) = True
isImm (Var    _ _) = True
isImm _            = False
```

We can now think of **immediate** expressions as:

```
type ImmExpr = {e:Expr | isImm e == True}
```

The *subset* of `Expr` *such that* `isImm` returns `True`

QUIZ

Similarly, lets write a function that describes **ANF** expressions

> *ANF means all* ***primitive operations*** *have* ***immediate*** *arguments.*

```
isAnf :: Expr a -> Bool
isAnf (Number  _     _) = True
isAnf (Var     _     _) = True
isAnf (Prim2 _ e1 e2 _) = _1
isAnf (If e1 e2 e3   _) = _2
isAnf (Let x e1 e2   _) = _3
```

What should we fill in for `_1`?

```
{- A -} isAnf e1
{- B -} isAnf e2
{- C -} isAnf e1 && isAnf e2
{- D -} isImm e1 && isImm e2
{- E -} isImm e2
```

QUIZ

Similarly, lets write a function that describes **ANF** expressions

> *ANF means all* ***primitive operations*** *have* ***immediate*** *arguments.*

```
isAnf :: Expr a -> Bool
isAnf (Number  _     _) = True
isAnf (Var     _     _) = True
isAnf (Prim1 _ e1 _)    = isAnf e1
isAnf (Prim2 _ e1 e2 _) = isImm e1 && isImm e2
isAnf (If e1 e2 e3   _) = isAnf e1 && isANF e2 && isANF e3
isAnf (Let x e1 e2   _) = isANF e1 && isANF e2
```

What should we fill in for `_2`?

```
{- A -} isAnf e1
{- B -} isImm e1
{- C -} True
{- D -} False
```

(but B also works)

We can now think of **ANF** expressions as:

```
type AnfExpr = {e:Expr | isAnf e == True}
```

The *subset* of `Expr` *such that* `isAnf` returns `True`

Use the above function to **test** our ANF conversion.

Types & Strategy

$e_1 + e_2$

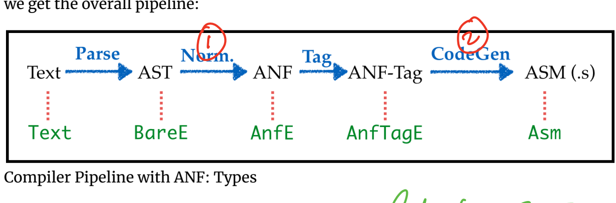
imm

x, y, z, ...
0, 1, 2, ...

Writing the type aliases:

```
type BareE   = Expr ()
type AnfE    = Expr ()   -- such that isAnf is True
type AnfTagE = Expr Tag  -- such that isAnf is True
type ImmTagE = Expr Tag  -- such that isImm is True
```

we get the overall pipeline:



Compiler Pipeline with ANF: Types

$(2+3)*(5-1) \longrightarrow$ let $t_1 = 2+3$
$t_2 = 5-1$
in $t_1 * t_2$

Transforms: Compiling `AnfTagE` to `Asm`

Compiler Pipeline: ANF to ASM

The compilation from ANF is easy, lets recall our examples and strategy:

Strategy: Given `v1 + v2` (where `v1` and `v2` are **immediate expressions**)

- Move `v1` into `rax`,
- Add `v2` to `rax`.

```
compile :: Env -> TagE -> Asm
compile env (Prim2 o v1 v2)
  = [ IMov      (Reg RAX) (immArg env v1)
    , (prim2 o) (Reg RAX) (immArg env v2)
    ]
```

where we have a helper to find the `Asm` variant of a `Prim2` operation

```
prim2 :: Prim2 -> Arg -> Arg -> Instruction
prim2 Plus  = IAdd
prim2 Minus = ISub
prim2 Times = IMul
```

and another to convert an *immediate expression* to an x86 argument:

```
immArg :: Env -> ImmTag -> Arg
immArg _   (Number n _) = Const n
immArg env (Var    x _) = RegOffset RBP i
  where
    i                   = fromMaybe err (lookup x env)
    err                 = error (printf "Error: '%s' is unbound" x)
```

QUIZ

Which of the below are in ANF ?

```
{- 1 -} 2 + 3 + 4

{- 2 -} let x = 12 in
          x + 1

{- 3 -} let x = 12
          , y = x + 6
        in
          x + y

{- 4 -} let x = 12
          , y = 18
          , t = x + y + 1
        in
          if t: 7 else: 9
```

- **A.** `1, 2, 3, 4`
- **B.** `1, 2, 3`
- **C.** `2, 3, 4`
- **D.** `1, 2`
- **E.** `2, 3`

Transforms: Compiling `Bare` to `Anf`

Next lets focus on **A-Normalization** i.e. transforming expressions into ANF

Compiler Pipeline: Bare to ANF

makeANF :: BareE → AnfE

A-Normalization

We can fill in the base cases easily

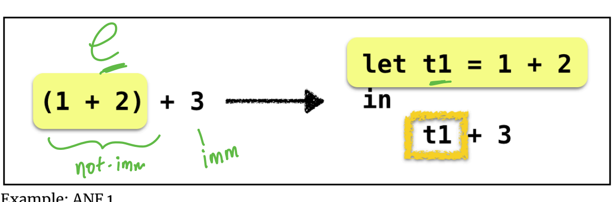
```
anf (Number n)      = Number n
anf (Var x)         = Var x
```

Interesting cases are the binary operations

Example: Anf-1

$e \longrightarrow (Id, ANF), Id$

Left operand is not immediate



Example: ANF 1

Key Idea: Helper Function

```
imm :: BareE -> ([(Id, AnfE)], ImmE)
```

`imm e` returns `([(t1, a1),...,(tn, an)], v)` where

- `ti, ai` are new temporary variables bound to ANF expressions
- `v` is an **immediate value** (either a constant or variable)

Such that `e` is *equivalent to*

```
let [t1 = a1
    , ...
    , tn = an ]
in
  v
```

$e \Rightarrow$

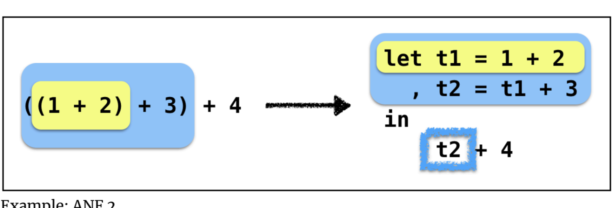
$e_1 + e_2 \Rightarrow$ let $x = mk\ e_1$, $y = mk\ e_2$ in $x + y$

Lets look at some more examples.

$e \Rightarrow t_1 = e_1 \ \vdots\ t_n = e_n$ v

Example: Anf-2

Left operand is not internally immediate



Example: ANF 2

IMM $(1+2)+3 \Longrightarrow [(t_1, 1+2), (t_2, t_1+3)]$

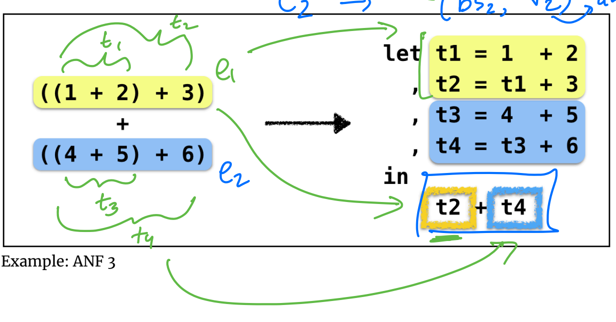
t_2

Example: Anf-3

$e_1 \Rightarrow (bs_1, v_1)$ aka t_2

$e_2 \Rightarrow (bs_2, v_2)$ aka t_4

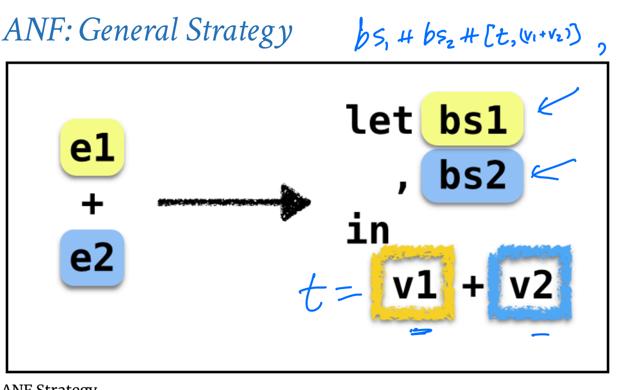
Both operands are not immediate



Example: ANF 3

ANF: General Strategy

$bs_1 +\!\!+\ bs_2 +\!\!+\ [t, (v_1+v_2)]$, t



ANF Strategy

1. **Invoke** `imm` on both the operands
2. **Concat** the `let` bindings
3. **Apply** the binary operator to the immediate values

Add1 (e) $[(t = add1(e'))]$

t

$e_1 + e_2$

ANF Implementation: Binary Operations

Lets implement the above strategy

```
anf (Prim2 o e1 e2) = lets (b1s ++ b2s)
                        (Prim2 o (Var v1) (Var v2))
  where
    (b1s, v1)       = imm e1
    (b2s, v2)       = imm e2

lets :: [(Id, AnfE)] -> AnfE -> AnfE
lets []         e' = e
lets ((x,e):bs) e' = Let x e (lets bs e')
```

Intuitively, `lets` *stitches* together a bunch of definitions:

```
lets [(x1, e1), (x2, e2), (x3, e3)] e
  ===> Let x1 e1 (Let x2 e2 (Let x3 e3 e))
```

(let x = 10
 in x + 5) + 4

e

e' = makeANF e

[(t, e')], t

ANF Implementation: Let-bindings

For `Let` just make sure we recursively `anf` the sub-expressions.

```
anf (Let x e1 e2)    = Let x e1' e2'
  where
    e1'              = anf e1
    e2'              = anf e2
```

$\underbrace{(2+3)}_{s_1} * \underbrace{(6-1)}_{s_2}$

ANF Implementation: Branches

Same principle applies to `If`

- use `anf` to recursively transform the branches.

```
anf (If e1 e2 e3) = If e1' e2' e3'
  where
    e1'             = anf e1
    e2'             = anf e2
    e3'             = anf e3
```

ANF: Making Arguments Immediate via `imm`

The workhorse is the function

```
imm :: BareE -> ([(Id, AnfE)], ImmE)
```

which creates temporary variables to crunch an arbitrary `Bare` into an *immediate* value.

No need to create an variables if the expression is *already* immediate:

```
imm (Number n l) = ( [], Number n l )
imm (Id     x l) = ( [], Id     x l )
```

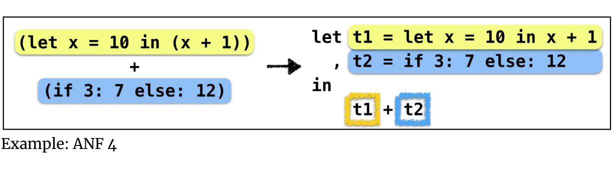
The tricky case is when the expression has a primitive operation:

```
imm (Prim2 o e1 e2) = ( b1s ++ b2s ++ [(t,  Prim2 o v1 v2)]
                      , Id t  )
  t                 = makeFreshVar ()
  (b1s, v1)         = imm e1
  (b2s, v2)         = imm e2
```

Oh, what shall we do when:

```
imm (If e1 e2 e3)   = ???
imm (Let x e1 e2)   = ???
```

Lets look at an example for inspiration.



Example: ANF 4

That is, simply

- `anf` the relevant expressions,
- bind them to a fresh variable.

```
imm e@(If _ _ _)  = immExp e
imm e@(Let _ _ _) = immExp e

immExp :: Expr -> ([(Id, AnfE)], ImmE)
immExp e = ([(t, e')], t)
  where
    e'   = anf e
    t    = makeFreshVar ()
```

One last thing: Whats up with `makeFreshVar` ?

Wait a minute, what is this magic **FRESH** ?

How can we create **distinct** names out of thin air?

(Sorry, no "global variables" in Haskell...)

We will use a counter, but will **pass its value around**

> *Just like doTag*

```
anf :: Int -> BareE -> (Int, AnfE)

anf i (Number n l)      = (i, Number n l)

anf i (Id     x l)      = (i, Id     x l)

anf i (Let x e b l)     = (i'', Let x e' b' l)
  where
    (i',  e')           = anf i e
    (i'', b')           = anf i' b

anf i (Prim2 o e1 e2 l) = (i'', lets (b1s ++ b2s) (Prim2 o e1' e2'
l))
  where
    (i' , b1s, e1')     = imm i  e1
    (i'', b2s, e2')     = imm i' e2

anf i (If c e1 e2 l)    = (i''', lets bs  (If c' e1' e2' l))
  where
    (i'  , bs, c')      = imm i   c
    (i'' ,     e1')     = anf i'  e1
    (i''',     e2')     = anf i'' e2
```

and

```
imm :: Int -> AnfE -> (Int, [(Id, AnfE)], ImmE)

imm i (Number n l)        = (i  , [], Number n l)

imm i (Var x l)           = (i  , [], Var x l)

imm i (Prim2 o e1 e2 l) = (i''', bs, Var v l)
  where
    (i'  , b1s, v1)     = imm i  e1
    (i'' , b2s, v2)     = imm i' e2
    (i''', v)           = fresh i''
    bs                  = b1s ++ b2s ++ [(v, Prim2 o v1 v2 l)]

imm i e@(If _ _ _  l)   = immExp i e

imm i e@(Let _ _ _ l)   = immExp i e

immExp :: Int -> BareE -> (Int, [(Id, AnfE)], ImmE)
immExp i e l  = (i'', bs, Var v ())
  where
    (i' , e') = anf i e
    (i'', v)  = fresh i'
    bs        = [(v, e')]
```

where now, the `fresh` function returns a *new counter* and a variable

```haskell
fresh :: Int -> (Int, Id)
fresh n = (n+1, "t" ++ show n)
```

Note this is super clunky. There *is* a really slick way to write the above code without the clutter of the `i` but thats too much of a digression, but feel free to look it up yourself

Recap and Summary

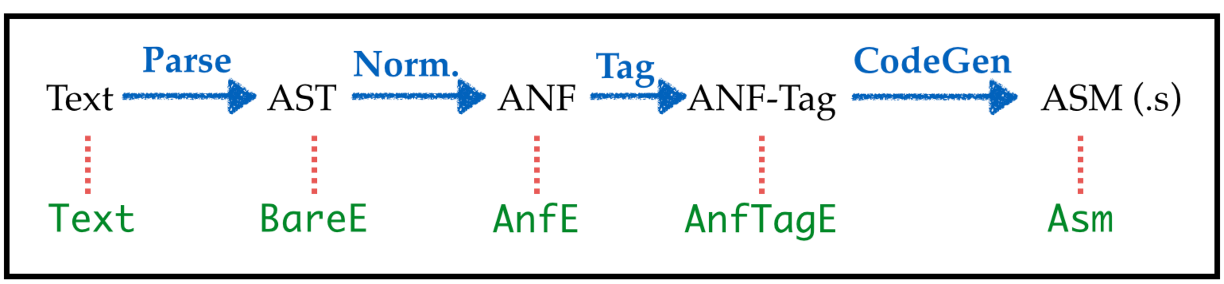
Just created `Boa` with

- Branches (`if` -expressions)
- Binary Operators (`+` , `-` , etc.)

In the process of doing so, we will learned about

- **Intermediate Forms**
- **Normalization**

Specifically,



Compiler Pipeline with ANF