

FDL "fer-de-lance"

First Class Functions

↳ Assign is up
due just
before final

fun Env
var Env

Functions as Values

Consider the following egg program

```
def f(it):
```

```
  it(5)
```

unbound func 'it'

it ∉ funenv

```
def incr(x):
```

```
  x + 1
```

```
f(incr)
```

unbound var 'incr'

incr ∉ varEnv

What will be the result of compiling/running?


on "diamond"

We have functions, but they are *second-class* entities in our languages: they don't have the same *abilities* as other values.

So, we get multiple error messages:


Errors found!

tests/input/hof.diamond:(2:3)-(4:1): Function 'it' is not defined



```
2|   it(5)
   ^ ^
```

tests/input/hof.diamond:7:3-7: Unbound variable 'incr'



```
7|   f(incr)
   ^ ^ ^ ^
```

This is because the **Env** only holds

- parameters, and
- let-bound variables

and **not** function definitions.

Functions as Values

But for the many reasons we saw in CSE 130 – we *want* to treat functions like values.
For example, if you run the above in Python you get:

```
>>> def f(it): return it(5)
```

```
>>> def incr(x): return x + 1
```

```
>>> f(incr)
```

```
6
```

Flashback: How do we compile `incr`?

We compile each function down into a sequence of instructions corresponding to its body.

```
def incr(x):
  x + 1
```

```
incr(5)
```

becomes, for incr

```
label_def_incr_start:
```

```
push rbp
```

```
mov rbp, rsp
```

```
mov rax, rdi
```

```
add rax, 2
```

```
mov rsp, rbp
```

```
pop rbp
```

```
ret
```

for the main expression

```
# setup stack frame
```

```
# grab param
```

```
# incr by 1
```

```
# undo stack frame
```

```
# buh-bye
```

our_code_here:

```
push rbp etc
```

```
⋮
```

```
mov rdi, 10
```

```
call label_def_incr_start:
```

```
⋮
```

```
pop rbp
etc
```

our_code_starts_here:

```
push rbp
```

```
mov rbp, rsp
```

```
mov rdi 10 # push arg '5'
```

```
call label_def_incr_start # call function
```

```
mov rsp, rbp
```

```
pop rbp
```

```
ret
```

What is the value of a function?

So now, lets take a step back. Suppose we want to compile

```
def f(it):
    it(5)
```

```
def incr(x):
    x + 1
```

```
f(incr)
```

*"enable passing
FUNCTION
as a param"*

*incr ↪ "label"
it ↪ ↗*

Attempt 1: What is the value of the parameter it ?

IDEA: Use the label where `incr` lives!

`label_def_f_start:`

```
push rbp
mov rbp, rsp
```

```
mov rax, rdi # grab function-address
mov rdi, 10 # push arg '5'
call rax # call function!
```

```
mov rsp, rbp
pop rbp
ret
```

```
def f(it):
    it(5)
```

- ① How do we know rdi is a func?
- ② what is the addr?

How to pass the value of the parameter ?

So now the main expression

`f(incr)`

can be compiled to:

our_code_starts_here:

```
push rbp
mov rbp, rsp
```

```
mov rdi, ?1    # push arg
call ?2       # call function
```

```
mov rsp, rbp
pop rbp
ret
```

*addr/label of
incr*

f(incr)

f(12)


func 'f' being called

QUIZ: What are suitable terms for ?1 and ?2 ?

	?1	?2
<input checked="" type="checkbox"/> A	label_def_incr_start	label_def_f_start
B	label_def_f_start	label_def_incr_start
C	label_def_f_start	label_def_f_start
D	label_def_incr_start	label_def_incr_start

Strategy Progression

1. **Representation = Start-Label**

- **Problem:** How to do run-time checks of valid args?
- 

Yay, that was easy! How should the following

behave?

```
def f(it):
    it(5)
```

```
def add(x, y):
    x + y
```

~~f(5)~~
f(add)

```
mov rax, L_add
mov rdi, 10
call rax
```

```
mov rax, rdi
add rax, rsi
gibberish
```

Lets see what Python does:

```
>>> def f(it): return it(5)
>>> def add(x,y): return x + y
>>> f(add)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 1, in f
TypeError: add() takes exactly 2 arguments (1 given)
```

Problem: Ensure Valid Number of Arguments?

How to make sure

- `f(incr)` **succeeds**, but
- `f(add)` **fails**

With **proper run-time error?**

1. **Where** does the run-time check happen?
2. What information is needed for the check?

`def f(it):`
`it(5)` — use metadata `f(incr)` ✓

`def incr(x):`
`x + 1` } create metadata `f(add)` ✗

`def add(x,y):`
`x + y`

Key: Need to *also* store the function's **arity**

- The **number of arguments** required by the function

Strategy Progression

1. Representation = ~~Start Label~~

- **Problem:** How to do run-time checks of valid args?

2. Representation = (Arity, Start-Label)

CREATE?

USE call

Attempt 2: What is the value of the parameter *it*?

```
def f(it):
    it(5)
```

$it \equiv \text{incr}$ → (1, l_incr_start)

```
def incr(x):
    x + 1
```

```
f(incr)
```

$it \equiv \text{add}$ → (2, l_add)

$\text{To } \begin{cases} 000 & \text{num} \\ 001 & \text{tuple} \\ 111 & \text{bool} \\ 101 & \text{fun-tuple} \end{cases}$

IDEA: Represent a function with a tuple of

$\{ \text{arity, function_start_label} \}$

- ✓ 1. check e is function
- ✓ 2. check args is correct
3. move args into reg/stack
4. call the function

We can now compile a call

$e(x_1, \dots, x_n)$

via the following strategy:

$\text{it}(5)$
=

$\text{rax} \leftarrow (e)$

1. $\text{assert}(\text{rax} \neq 0x111 \equiv \&101)$
2. $\text{assert}(\text{len } xs == \text{rax}[0])$
3. as before
4. call $(\text{rax}[1])$

- ✓ 1. **Evaluate** the tuple e
- ✓ 2. **Check** that $e[0]$ is equal to n (else arity mismatch error)
- ✓ 3. **Call** the function at $e[1]$

Example

Lets see how we would compile this

```
def f(it):
  it(5)
```

```
def incr(x):
  x + 1
```

```
f(incr)
```

We need to map the variable

incr

to the tuple

```
(1, label_def_incr_start)
```

let $f = \lambda it \rightarrow it(5)$

, $incr = \lambda x \rightarrow x + 1$

in

$f(incr)$

1. create "tuple" for incr

2. name 'incr'

put the func-tuples
IN the var env

But **where** will we store this information?

Strategy Progression

1. **Representation** = Start-Label

- **Problem:** How to do run-time checks of valid args?

2. **Representation** = (Arity, Start-Label)



- **Problem:** How to map function **names** to tuples?

3. **Lambda Terms** Make functions just another expression!

Attempt 3: Lambda Terms

So far, we could only define functions at **top-level**

- First-class functions are like *any* other expression,
- Can define a function, wherever you have any other expression.

Language	Syntax
Haskell	$\lambda(x_1, \dots, x_n) \rightarrow e$ 
Ocaml	<code>fun (x1, ..., xn) -> e</code> 

Language	Syntax
JS	$(x_1, \dots, x_n) \Rightarrow \{ \text{return } e \}$
C++	$[\&](x_1, \dots, x_n) \{ \text{return } e \}$

Example: Lambda Terms

We can now replace `def` as:

```
let f = (lambda (it): it(5))  
    , incr = (lambda (x): x + 1)  
in  
  f(incr)
```

Implementation

As always, to the details! Lets figure out:

Representation

1. How to store **function-tuples**



Types:

1. ~~Remove Def~~
2. Add lambda to Expr

Transforms

1. Update tag and ANF ✓
2. Update checker ✓
3. Update compile ✓

LAM
APP ✓

Implementation: Representation

Represent 'lambda-tuples' or function-tuples'' via a special tag:

Type	LSB
number	xx0
boolean	111
pointer	001
function	101

In our code:

```
data Ty = ... | TClosure
```

```
typeTag :: Ty -> Arg
```

```
typeTag TTuple    = HexConst 0x00000001
```

```
typeTag TClosure  = HexConst 0x00000005
```

```
typeMask :: Ty -> Arg
```

```
typeMask TTuple    = HexConst 0x00000007
```

```
typeMask TClosure  = HexConst 0x00000007
```

3 LSB

So, **Function Values** represented just like a tuples

- padding, ~~52~~ etc.
- but with tag 101.

Crucially, we can get 0 -th, or 1 -st elements from tuple.

Question: Why not use plain tuples?

↳ to ensure e[1] is
a func label!

Implementation: Types

First, lets look at the new Expr type

- ~~No more Def~~

`data Expr a`
`= ...`
`| Lam [Bind a] !(Expr a) a -- fun. definitions`
`| App !(Expr a) [Expr a] a -- fun. calls`

params (arrow pointing to `[Bind a]`)
body (arrow pointing to `!(Expr a) a`)

So we represent a **function-definition** as:

`Lam [x1, ..., xn] e`

Lam [x₁, ..., x_n] e

and a **function call** as:

`App e [e1, ..., en]`

App e [e₁, ..., e_n]

previously D which we lookup in FunEnv

Transforms: Tag

This is pretty straight forward (do it yourself)

Transforms: ANF

QUIZ:

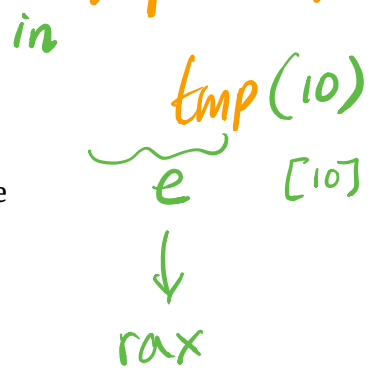
(Apples)

Does e need to be

1 n

let tup =
 (lambda(x) : x+1
 ; lambda(y) : y-1
)
 tmp = tup[0]

- A Immediate
- B ANF
- C None of the above



Transforms: ANF

QUIZ:

(App e es)

Do es need to be

- A Immediate
- B ANF
- C None of the above

Transforms: ANF

The App case, fun + args should be immediate

- **Need the values to push on stack and make the call happen!**

Just like function calls (in diamondback), except

- Must also handle the **callee-expression** (named *e* below)

$\text{anf } i \text{ (App } e \text{ es)} = (i', \text{ stitch } bs \text{ (App } v \text{ vs)})$

where

$(i', bs, v:vs) = \text{imms } i \text{ (e:es)}$

Transforms: ANF

QUIZ:

$(\text{Lam } xs \text{ } e)$

Does e need to be

(A) IMM
 (B) ANF is OK

- **A** Immediate
- **B** ANF
- **C** None of the above

Transforms: ANF

The `Lam` case, the body will be **executed** (when called)

- So we just need to make sure its in ANF (like all the code!)

$\text{anf } i \text{ (Lam } xs \text{ e)} = (i', \text{Lam } xs \text{ e}')$

where

$(\underline{i'}, \underline{e'}) = \text{anf } i \text{ e}$

Transforms: Checker

We just have Expr (no Def) so there is a single function:

```
wellFormed :: BareExpr -> [UserError]
```

```
wellFormed = go emptyEnv
```

```
where
```

```
  gos = concatMap . go
```

```
  go _ (Boolean {}) = ... ✓
```

```
  go _ (Number n l) = largeNumberErrors n l
```

```
  go vEnv (Id x l) = unboundVarErrors vEnv x l
```

```
  go vEnv (Prim1 _ e _) = ... ✓
```

```
  go vEnv (Prim2 _ e1 e2 _) = ... ✓
```

```
  go vEnv (If e1 e2 e3 _) = ... ✓
```

```
  go vEnv (Let x e1 e2 _) = ... ++ go vEnv e1 ++ go (addEnv x vEnv
```

```
v) e2
```

```
  go vEnv (Tuple es _) = ...
```

```
  go vEnv (GetItem e1 e2 _) = ...
```

```
  go vEnv (App e es _) = ?1 gos env (e:es)
```

```
  go vEnv (Lam xs e _) = ?2 ++ go ?3 e
```

- How shall we implement ?1 ?
- How shall we implement ?2 ?
- How shall we implement ?3 ?

① 't' even in scope
 ② ~~'e' has arity 1~~

int → int

let $f = \lambda it \rightarrow \boxed{it(5)}$

, $incr = \lambda x \rightarrow x+1$

, $add = \lambda x y \rightarrow x+y$

in ... $\boxed{f(incr)}$ $\boxed{f(add)}$